E−Companion

Appointment Scheduling under Patient Preference and No-Show Behavior

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A Appendix: Model Extensions

In this section, we describe a number of extensions of our approach that can be used to formulate certain dimensions of the appointment scheduling problem at a more detailed level.

A.1 Generalization of the Multinomial Logit Choice Model

Multinomial logit model assumes that the preference weight associated with one alternative does not depend on the offer set. However, in the appointment scheduling context, it is possible that patients place a higher value on the choice of seeking care elsewhere when too few appointment choices are offered. To capture this dependence, we can use a general multinomial logit model introduced by Gallego et al. (2011). In this extended choice model, each day in the scheduling horizon has two parameters, denoted by v_j and w_j. If we offer the subset S of days, then a patient chooses day j in the scheduling horizon with probability $P_j(S) = v_j/(1 + \sum_{k \notin S} w_k + \sum_{k \in S} v_k)$, where the quantity $\sum_{k \notin S} w_k$ captures the increase in the preference weight of not booking an appointment as a function of the days that are not offered. All of our results in the paper continue to hold under this more general form of the multinomial logit model.

A.2 Capturing Heterogeneity in Patient Choice Behavior

Our approach implicitly assumes that the choice behavior represented by the multinomial logit model captures the overall patient population preferences. This approach is reasonable either when the patient population is homogeneous in its preference profile, or if we do not have access to additional information that can classify patients in terms of their preferences at time of making scheduling decisions. When additional information is available, subset offer decisions may depend on such information. For example, considering our computational study, if patients having urgent and ambiguous conditions arrive simultaneously into the system and we have information about the condition of the patient before offering the available appointment days, then we can define two sets of decision variables $h^U = \{h^U(S) : S \subset \mathcal{T}\}$ and $h^A = \{h^A\{S) : S \subset \mathcal{T}\}$ to respectively capture the probability that each subset of days is offered to a patient with urgent and ambiguous conditions. If we make the subset offer decisions with information about the patient condition, then we can use an approach similar to the one in this paper and transform a static model with exponentially many subset offer probabilities into an equivalent static model whose decision variables grows only linearly in the length of the scheduling horizon. Furthermore, building on this static model, we can also formulate a dynamic model by using one step of the policy improvement algorithm.

A.3 Capturing Day-of-the-Week Effect on Preferences

In our static model, we assume that the demand is stationary and the preference of a patient for different appointment days depends on how many days into the future the appointment is made, rather than the particular day of the week of the appointment. This assumption allowed us to focus on the expected profit per day in problem (2)-(4). In practice, however, the patient demand may be nonstationary, depending on the day of the week. Furthermore, the preference of a patient for different days of the week may be as pronounced as the preference for different appointment delays. It is possible to generalize our model so that both of these effects are captured.

For instance, assume that nonstationarities follow a weekly pattern and the scheduling horizon τ is a multiple of a week. In this case, we can work with the extended set of decision variables by letting $h_t(S)$ be the probability with which we offer the subset S of days given that we are on day t of the week. Using these decision variables, we can construct a model analogous to problem (2)-(4), but we account for the total expected profit per week, rather than expected profit per day. As an example, consider a clinic that is open 7 days a week and is using an appointment window of 14 days. Without loss of generality, we consider the week consisting of days 15 to 21, so the earliest day when patients can be schedule into this week is day 1. Adapting the notation used earlier to the non-stationary setting, we let λ_t denote the mean demand for day t.

Let $\mathcal{T}_t = \{t, t+1, \ldots, t+14\}$ represent the set of days in which patients arriving on day t can be scheduled. The decision variable in this model is $h_t(S)$, which represents the probability of offering the subset of days $S \subset \mathcal{T}_t$ on day t. Note that the offering set probability distribution for future days are nonstationary as well and depends on the day of the week. To be concise, we write the set $\{j-7 : j \in S\}$ as $S-7$, i.e., the set $S-7$ is obtained by subtracting every element in S by 7 days. Then, $h_t(S) = h_{t-7}(S-7)$. Let $P_t(S)$ be the probability that an arriving patient will choose day t when offered set S. Then, for a patient who arrives on day $t-j$, the probability that she will choose day t is given by $\sum_{S \subset \mathcal{T}_{t-j}} P_t(S) h_{t-j}(S)$. Thus the number of patients who schedule an appointment to day t and are retained until the morning of day t is given by a Poisson random variable with mean $\sum_{j=0}^{14} \lambda_{t-j} \bar{r}_j \sum_{S \subset \mathcal{T}_{t-j}} P_t(S) h_{t-j}(S)$. Similarly, the number of patients who actually show up on day t is also a Poisson random variable with mean $\sum_{j=0}^{14} \lambda_{t-j} \bar{s}_j \sum_{S \subset \mathcal{T}_{t-j}} P_t(S) h_{t-j}(S)$. Then, we can formulate a static model as

$$
\begin{aligned}
\max \quad & \sum_{t=15}^{21} \sum_{j=0}^{14} \lambda_{t-j} \bar{s}_j \sum_{S \subset \mathcal{T}_{t-j}} P_t(S)\, h_{t-j}(S) \\
& \qquad - \theta \sum_{t=15}^{21} \mathbb{E}\Big\{ \Big[\mathsf{Pois}\Big(\sum_{j=0}^{14} \lambda_{t-j} \bar{r}_j \sum_{S \subset \mathcal{T}_{t-j}} P_t(S)\, h_{t-j}(S) \Big) - C \Big]^+ \Big\} \\
\text{subject to} \quad & \sum_{S \subset \mathcal{T}_t} h_t(S) = 1 \qquad \forall\, t \in \{1, 2, \ldots, 21\} \\
& h_t(S) = h_{t-7}(S-7) \qquad \forall\, t \in \{8, 9, \ldots, 21\},\ \forall\, S \subset \mathcal{T}_t \\
& h_t(S) \geq 0, \qquad \forall\, t \in \{1, 2, \ldots, 21\},\ \forall\, S \subset \mathcal{T}_t.
\end{aligned}
$$

The first constraint ensures that the probability we offer a set of days in any given day is one, whereas the second constraint captures the weekly pattern of decision variables. For this extended version of problem (2)-(4), we can still come up with a transformation similar to the one in Section

4.1 that reduces the number of decision variables from exponential in the length of the scheduling horizon to only linear.

Finally, another simplifying assumption our model makes is that while patients have preferences for which day of the week they would like to be seen they do not have any preferences for the specific appointment time of the day. It is important to note that the same way we formulate preferences on different days of the week (as shown above), we can also incorporate preferences for different times of the day, as long as we make the assumption that the expected cost the clinic incurs on a given day only depends on the total number of patients scheduled for that day, but not the specific times of the appointments.

B Appendix: Omitted Results

In this section, we give the proofs of the results that are omitted in the paper.

B.1 Proof of Proposition 1

We complete the proof of Proposition 1 in two parts. First, assume that $h^* = \{h^*(S) : S \subset \mathcal{T}\}$ is an optimal solution to problem (2)-(4). Letting $x_j^* = \sum_{S \subset \mathcal{T}} P_j(S)\, h^*(S)$ and $u^* = \sum_{S \subset \mathcal{T}} N(S)\, h^*(S)$, we need to show that (x^*, u^*) is a feasible solution to problem (5)-(8) providing the same objective value as the solution h^*. We have

$$\sum_{j \in \mathcal{T}} x_j^* + u^* = \sum_{S \subset \mathcal{T}} h^*(S)[\sum_{j \in \mathcal{T}} P_j(S) + N(S)] = \sum_{S \subset \mathcal{T}} h^*(S) = 1,$$

where the second equality follows by the definition of the multinomial logit model and the third equality follows since h^* is feasible to problem (2)-(4). Thus, the solution (x^*, u^*) satisfies the first constraint in problem (5)-(8). On the other hand, using $\mathbf{1}(\cdot)$ to denote the indicator function, we have $x_j^*/v_j = \sum_{S \subset \mathcal{T}} [\mathbf{1}(j \in S)\, h^*(S)/(1 + \sum_{k \in S} v_k)]$ by the definition of $P_j(S)$ in the multinomial logit model. Noting that $u = \sum_{S \subset \mathcal{T}} [h^*(S)/(1 + \sum_{k \in S} v_k)]$, it follows that $x_j^*/v_j \le u$, indicating that the solution (x^*, u^*) satisfies the second set of constraints in problem (5)-(8). Since $x_j^* = \sum_{S \subset \mathcal{T}} P_j(S)\, h^*(S)$, comparing the objective functions of problems (2)-(4) and (5)-(8) shows that the solutions h^* and (x^*, u^*) provide the same objective values for their problems.

Second, assume that (x^*, u^*) is a feasible solution to problem (5)-(8). We construct the solution h^* as in (9). To see that the solutions (x^*, u^*) and h^* provide the same objective values for their respective problems, we observe that

$$\sum_{S \subset \mathcal{T}} P_j(S)\, h^*(S) = \sum_{i=0}^{\tau} P_j(S_i)\, h^*(S_i) = \sum_{i=j}^{\tau} P_j(S_i)\, h^*(S_i) = \sum_{i=j}^{\tau} v_j \Big[\frac{x_i^*}{v_i} - \frac{x_{i+1}^*}{v_{i+1}} \Big] = x_j^*,$$

where the first equality is by the fact that $h^*(S)$ takes positive values only for the sets $S_0, \ldots, S_\tau$ and $\emptyset$, the second equality is by the fact that $j \in S_i$ only when $j \le i$ and the third equality follows by the definition of $h^*(S_i)$ and noting that $x^*_{\tau+1} = 0$. Using the equality above and comparing the objective functions of problems (2)-(4) and (5)-(8) show that the solutions h^* and (x^*, u^*) provide the same objective values for their problems. To see that the solution h^* is feasible to problem (2)-(4), we let $V_i = 1 + \sum_{k \in S_i} v_k$ for notational brevity and write $\sum_{S \subset \mathcal{T}} h^*(S)$ as

$$u^* - \frac{x^*_0}{v_0} + \sum_{j=0}^{\tau} V_j \Big[\frac{x^*_j}{v_j} - \frac{x^*_{j+1}}{v_{j+1}}\Big] = u^* + (V_0 - 1)\frac{x^*_0}{v_0} + (V_1 - V_0)\frac{x^*_1}{v_1} + \ldots + (V_\tau - V_{\tau-1})\frac{x^*_\tau}{v_\tau} = 1,$$

where the first equality follows by rearranging the terms and using the convention that $x^*_{\tau+1} = 0$ and the second equality is by noting that $V_i - V_{i-1} = v_i$ and using the fact that (x^*, u^*) is feasible to problem (5)-(8) so that $\sum_{j=0}^{\tau} x^*_j + u^* = 1$. □

B.2 Lemma 7

The following lemma is used in Section 4.

Lemma 7. *Letting $F(\alpha) = \mathbb{E}\big\{[\mathsf{Pois}(\alpha) - C]^+\big\}$, $F(\cdot)$ is differentiable and convex.*

Proof. The proof uses elementary properties of the Poisson distribution. By using the probability mass function of the Poisson distribution, we have

$$\begin{aligned} F(\alpha) &= \sum_{i=C+1}^{\infty} \frac{e^{-\alpha}\,\alpha^i}{i!}\,(i - C) = \sum_{i=C+1}^{\infty} \frac{e^{-\alpha}\,\alpha^i}{(i-1)!} - \sum_{i=C+1}^{\infty} \frac{e^{-\alpha}\,\alpha^i}{i!}\,C \\ &= \alpha \sum_{i=C}^{\infty} \frac{e^{-\alpha}\,\alpha^i}{i!} - C \sum_{i=C+1}^{\infty} \frac{e^{-\alpha}\,\alpha^i}{i!} = \alpha\,\mathbb{P}\{\mathsf{Pois}(\alpha) \ge C\} - C\,\mathbb{P}\{\mathsf{Pois}(\alpha) \ge C+1\}. \end{aligned} \tag{25}$$

Thus, the differentiability of $F(\cdot)$ follows by the differentiability of the cumulative distribution function of the Poisson distribution with respect to its mean. For the convexity of $F(\cdot)$, we have

$$\begin{aligned} \frac{d\,\mathbb{P}\{\mathsf{Pois}(\alpha) \ge C\}}{d\alpha} &= -\frac{d\,\mathbb{P}\{\mathsf{Pois}(\alpha) \le C-1\}}{d\alpha} = -\sum_{i=0}^{C-1} \frac{d\Big(\frac{e^{-\alpha}\,\alpha^i}{i!}\Big)}{d\alpha} \\ &= \sum_{i=0}^{C-1} \frac{e^{-\alpha}\,\alpha^i}{i!} - \sum_{i=1}^{C-1} \frac{e^{-\alpha}\,\alpha^{i-1}}{(i-1)!} = \mathbb{P}\{\mathsf{Pois}(\alpha) = C-1\}. \end{aligned}$$

In this case, if we differentiate both sides of (25) with respect to α and use the last chain of equalities, then we obtain

$$\begin{aligned} \frac{dF(\alpha)}{d\alpha} &= \mathbb{P}\{\mathsf{Pois}(\alpha) \ge C\} + \alpha\,\mathbb{P}\{\mathsf{Pois}(\alpha) = C-1\} - C\,\mathbb{P}\{\mathsf{Pois}(\alpha) = C\} \\ &= \mathbb{P}\{\mathsf{Pois}(\alpha) \ge C\} + \alpha\,\frac{e^{-\alpha}\,\alpha^{C-1}}{(C-1)!} - C\,\frac{e^{-\alpha}\,\alpha^{C}}{C!} = \mathbb{P}\{\mathsf{Pois}(\alpha) \ge C\}. \end{aligned}$$

To see that $F(\cdot)$ is convex, we use the last two chains of equalities to observe that the second derivative of $F(\alpha)$ with respect to α is $\mathbb{P}\big\{\mathsf{Pois}(\alpha) = C-1\big\}$, which is positive. □

B.3 Proof of Corollary 3

By Proposition 2, there exists an optimal solution x^* to problem (5)-(8) that satisfies (16). We define the subsets $S_0, S_1, \ldots, S_\tau$ as in the proof of Proposition 1 and construct an optimal solution h^* to problem (2)-(4) by using x^* as in (9). In this case, since x^* satisfies (16) for some $k \in \mathcal{T}$, only two of the decision variables $\{h(S) : S \subset \mathcal{T}\}$ can take on nonzero values and these two decision variables are $h^*(S_{k-1})$ and $h^*(S_k)$. Thus, the desired result follows by observing that S_k is a subset of the form $\{0, 1, \ldots, k\}$. $\Box$

B.4 Proof of Lemma 4

We let $\pi^*(S)$ be the steady state probability with which we offer the subset S of days under the optimal, possibly state-dependent, policy. So, if we consider a particular day in steady state, then the number of patients that are scheduled for this day j days in advance is given by a Poisson random variable with mean $\sum_{S \subset \mathcal{T}} \lambda \, P_j(S) \, \pi^*(S)$. Therefore, if we use the random variable A_j^* to denote the number of patients that we schedule for a particular day j days in advance in steady state, then A_j^* has mean $\sum_{S \subset \mathcal{T}} \lambda \, P_j(S) \, \pi^*(S)$. We note that $A_1^*, A_2^*, \ldots, A_\tau^*$ are not necessarily independent of each other, since the decisions under the optimal state-dependent policy on different days can be dependent. Similarly, in steady state, we let S_j^* be the number of patients that we schedule for a particular day j days in advance and that show up under the optimal state-dependent policy. Finally, we let R_j^* be the number of patients that we schedule for a particular day j days in advance and that we retain until the morning of the appointment under the optimal state dependent policy. Noting that the show-up and cancellation decisions of the patients are independent of how many patients we schedule for a particular day, we have $\mathbb{E}\{S_j^*\} = \bar{s}_j \, \mathbb{E}\{A_j^*\}$ and $\mathbb{E}\{R_j^*\} = \bar{r}_j \, \mathbb{E}\{A_j^*\}$. In this case, the average profit per day generated by the optimal state-dependent policy satisfies

$$\begin{aligned} V^* = \mathbb{E}\Big\{ \sum_{j \in \mathcal{T}} S_j^* \Big\} - \theta \, \mathbb{E}\Big\{ \Big[\sum_{j \in \mathcal{T}} R_j^* - C \Big]^+ \Big\} &\leq \sum_{j \in \mathcal{T}} \mathbb{E}\{S_j^*\} - \theta \Big[\sum_{j \in \mathcal{T}} \mathbb{E}\{R_j^*\} - C \Big]^+ \\ &= \sum_{j \in \mathcal{T}} \sum_{S \subset \mathcal{T}} \lambda \, \bar{s}_j \, P_j(S) \, \pi^*(S) - \theta \Big[\sum_{j \in \mathcal{T}} \sum_{S \subset \mathcal{T}} \lambda \, \bar{r}_j \, P_j(S) \, \pi^*(S) - C \Big]^+ \leq Z_{DET}. \end{aligned}$$

In the chain of inequalities above, the first inequality is by the Jensen's inequality. The second equality is by $\mathbb{E}\{S_j^*\} = \bar{s}_j \, \mathbb{E}\{A_j^*\}$ and $\mathbb{E}\{R_j^*\} = \bar{r}_j \, \mathbb{E}\{A_j^*\}$. To see the second inequality, we note that $\{\pi^*(S) : S \subset \mathcal{T}\}$ is a feasible but not necessarily an optimal solution to the problem

$$\begin{aligned} \max \quad & \sum_{j \in \mathcal{T}} \sum_{S \subset \mathcal{T}} \lambda \, \bar{s}_j \, P_j(S) \, w(S) - \theta \Big[\sum_{j \in \mathcal{T}} \sum_{S \subset \mathcal{T}} \lambda \, \bar{r}_j \, P_j(S) \, w(S) - C \Big]^+ \\ \text{subject to} \quad & \sum_{S \subset \mathcal{T}} w(S) = 1 \\ & w(S) \geq 0 \qquad S \subset \mathcal{T} \end{aligned}$$

and the optimal objective values of the problem above and problem (20)-(23) are equal to each other, which can be verified by using the argument in the proof of Proposition 1. $\Box$

B.5 Proof of Proposition 5

Letting $(\hat{x}, \hat{u})$ be an optimal solution to problem (20)-(23), we have $\Pi(x^*) \geq \Pi(\hat{x})$. Since we can always offer the empty set with probability one, the optimal objective value of problem (5)-(8) is nonnegative and we obtain $\Pi(x^*)/V^* = [\Pi(x^*)]^+/V^* \geq [\Pi(\hat{x})]^+/V^*$. Using Lemma 4, we continue this chain of inequalities as $[\Pi(\hat{x})]^+/V^* \geq [\Pi(\hat{x})]^+/Z_{DET} \geq \Pi(\hat{x})/Z_{DET} = 1 - (Z_{DET} - \Pi(\hat{x}))/Z_{DET}$. So, it is enough to show that the second term on the right side of (24) upper bounds $(Z_{DET} - \Pi(\hat{x}))/Z_{DET}$. For a Poisson random variable with mean α, we claim that

$$\mathbb{E}\{[\mathsf{Pois}(\alpha) - C]^+\} \leq [\alpha - C]^+ + \alpha/\sqrt{2\,\pi\,C}. \tag{26}$$

This claim will be proved at the end. With this claim and letting $\beta = \sum_{j=1}^{T} \lambda\, \bar{s}_j\, \hat{x}_j$ and $\alpha = \sum_{j=1}^{T} \lambda\, \bar{r}_j\, \hat{x}_j$ for notational brevity, we obtain

$$\frac{Z_{DET} - \Pi(\hat{x})}{Z_{DET}} = \frac{[\beta - \theta\,[\alpha - C]^+] - [\beta - \theta\,\mathbb{E}\{[\mathsf{Pois}(\alpha) - C]^+\}]}{Z_{DET}} \leq \frac{\frac{\theta\,\alpha}{\sqrt{2\,\pi\,C}}}{Z_{DET}} \leq \frac{\frac{\theta\,\lambda\,\bar{r}_0}{\sqrt{2\,\pi\,C}}}{Z_{DET}}, \tag{27}$$

where the second inequality is by noting that $\sum_{j \in \mathcal{T}} \hat{x}_j \leq 1$, $\bar{r}_0 \geq \bar{r}_1 \geq \ldots \geq \bar{r}_\tau$ so that $\alpha \leq \lambda\,\bar{r}_0$.

We proceed to constructing a lower bound on Z_{DET}. The solution $(\tilde{x}, \tilde{u})$ we obtain by setting $\tilde{x}_0 = \frac{v_0}{1+v_0}$, $\tilde{u} = \frac{1}{1+v_0}$ and all other decision variables to zero is feasible to problem (20)-(23). Thus, if $\lambda\,\bar{r}_0\,\frac{v_0}{1+v_0} \leq C$, then we can lower bound Z_{DET} as $Z_{DET} \geq \lambda\,\bar{s}_0\,\tilde{x}_0 - \theta\,[\lambda\,\bar{r}_0\,\tilde{x}_0 - C]^+ = \lambda\,\bar{s}_0\,\frac{v_0}{1+v_0}$. On the other hand, if $\lambda\,\bar{r}_0\,\frac{v_0}{1+v_0} > C$, then the solution $(\tilde{x}, \tilde{u})$ we obtain by setting $\tilde{x}_0 = \frac{C}{\lambda\,\bar{r}_0}$, $\tilde{u} = 1 - \frac{C}{\lambda\,\bar{r}_0}$ and all other decision variables to zero is feasible to problem (20)-(23). Thus, if $\lambda\,\bar{r}_0\,\frac{v_0}{1+v_0} > C$, then we can lower bound Z_{DET} as $Z_{DET} \geq \lambda\,\bar{s}_0\,\tilde{x}_0 - \theta\,[\lambda\,\bar{r}_0\,\tilde{x}_0 - C]^+ = \bar{s}_0\,\frac{C}{\bar{r}_0}$. Collecting the two cases together, we lower bound Z_{DET} by $\bar{s}_0 \min\left\{\lambda\,\frac{v_0}{1+v_0}, \frac{C}{\bar{r}_0}\right\}$. Continuing the chain of inequalities in (27) by using the lower bound on Z_{DET}, we obtain

$$\frac{\frac{\theta\,\lambda\,\bar{r}_0}{\sqrt{2\,\pi\,C}}}{Z_{DET}} \leq \frac{\frac{\theta\,\lambda\,\bar{r}_0}{\sqrt{2\,\pi\,C}}}{\bar{s}_0 \min\left\{\lambda\,\frac{v_0}{1+v_0}, \frac{C}{\bar{r}_0}\right\}}.$$

Arranging the terms in the last expression above yields the desired result.

Finally, we prove the claim (26) made above. For $k \geq C+1$, we observe that $[k-C]^+ - [\alpha-C]^+ \leq k - \alpha$. In particular, for $k \geq \alpha$, this inequality follows by the Lipschitz continuity of the function $[\cdot - C]^+$. For $k < \alpha$, we have $C+1 \leq k < \alpha$ and it follows $[k-C]^+ - [\alpha - C]^+ = k - \alpha$, establishing the desired inequality. In this case, the result in the lemma follows by noting that

$$\begin{aligned}\mathbb{E}\{[\mathsf{Pois}(\alpha) - C]^+\} &= \sum_{k=C+1}^{\infty} [k - C]^+\,\frac{e^{-\alpha}\alpha^k}{k!} \leq [\alpha - C]^+ + \sum_{k=C+1}^{\infty} \big[[k-C]^+ - [\alpha - C]^+\big]\,\frac{e^{-\alpha}\alpha^k}{k!} \\ &\leq [\alpha - C]^+ + \sum_{k=C+1}^{\infty} (k - \alpha)\frac{e^{-\alpha}\alpha^k}{k!} = [\alpha - C]^+ + \frac{e^{-\alpha}\alpha^C}{C!}\,\alpha \leq [\alpha - C]^+ + \frac{e^{-C}C^C}{C!}\,\alpha,\end{aligned}$$

where the first inequality follows by adding and subtracting $[\alpha - C]^+$ to the expression on the left side of this inequality, the second inequality follows by the inequality derived at the beginning of the proof, the last equality is by arranging the terms in the summation on the left side of this inequality and the third inequality is by noting that the function $f(\alpha) = e^{-\alpha}\alpha^C$ attains its maximum at $\alpha = C$. In this case, the result follows by noting that $C! \geq \sqrt{2\,\pi\,C}(C/e)^C$ by Stirling's approximation and using this bound on the right side of the chain of inequalities above. $\square$

B.6 Proof of Proposition 6

The proof follows from an argument similar to the one in the proof of Proposition 1. Assume that $h^* = \{h^*(S) : S \subset \mathcal{T}\}$ is an optimal solution to problem (2)-(4) with the additional constraints $h_t(S) \in \{0,1\}$ for all $S \subset \mathcal{T}$. Letting $x_j^* = \sum_{S\subset\mathcal{T}} P_j(S)\,h^*(S)$ and $u^* = \sum_{S\subset\mathcal{T}} N(S)\,h^*(S)$, we can follow the same argument in Section B.1 to show that (x^*, u^*) with $x^* = (x_0^*, \ldots, x_\tau^*)$ is a feasible solution to problem (5)-(8) with the additional constraints $x_j/v_j \in \{0, u\}$ for all $j \in \mathcal{T}$. Furthermore, the objective values provided by the two solutions for their respective problems are identical. On the other hand, assume that (x^*, u^*) with $x^* = (x_0^*, \ldots, x_\tau^*)$ is an optimal solution to problem (5)-(8) with the additional constraints $x_j/v_j \in \{0, u\}$ for all $j \in \mathcal{T}$. We reorder and reindex the days in the scheduling horizon so that we have $u^* = x_0^*/v_0 = x_1^*/v_1^* = \ldots = x_{j-1}^*/v_{j-1}^* \geq x_j^*/x_j = x_{j+1}^*/v_{j+1} = \ldots = x_\tau^*/v_\tau^* = 0$. We define the subsets $S_0, S_1, \ldots, S_\tau$ as $S_j = \{0, 1, \ldots, j\}$. For notational convenience, we define $x_{\tau+1}^* = 0$. In this case, letting

$$h^*(\emptyset) = u^* - \frac{x_0^*}{v_0} \quad \text{and} \quad h^*(S_j) = \Big[1 + \sum_{k\in S_j} v_k\Big]\Big[\frac{x_j^*}{v_j} - \frac{x_{j+1}^*}{v_{j+1}}\Big]$$

for all $j = 0, 1, \ldots, \tau$ and letting $h^*(S) = 0$ for all other subsets of $\mathcal{T}$, we can follow the same argument in Section B.1 to show that $\{h^*(S) : S \subset \mathcal{T}\}$ is a feasible solution to problem (2)-(4) with the additional constraints $h_t(S) \in \{0,1\}$ for all $S \subset \mathcal{T}$. Furthermore, we can check that the two solutions provide the same objective value for their respective problems. $\square$

B.7 Sample Survey Questions

<u>Q 3.</u> For the following questions, imagine that you are in your current state of health but over the last few months you have been feeling tired and irritable and have had difficulty sleeping. You have tried several things yourself to remedy this but are not feeling any better. You decide to seek a medical opinion from doctors at the Farrell clinic. Now you are making calls to Farrell to schedule an appointment.

If the clinic can only offer you an appointment <u>today or 2 days from now</u>, ***considering your current work/life schedule***, which appointment date would you take or would you seek care elsewhere, e.g., try to find another physician or go to an emergency room? Choose <u>one</u> option below by placing an "x" in the box that best represents your response.

☐ Today

☐ 2 Days from Now

☐ Seek Care Elsewhere

Figure 1: Sample choice question under ambiguous health condition.

<u>Q 7.</u> For the following questions, imagine that you have a heavy cough and cold today. Over the past 2 days you have started to get some pain in the right side of your chest. It is very sharp and worse if you cough or take a deep breath in. You decide to seek a medical opinion from doctors at the Farrell clinic. Now you are making calls to Farrell to schedule an appointment.

If the clinic can only offer you an appointment <u>today or 2 days from now</u>, ***considering your current work/life schedule***, which appointment date would you take or would you seek care elsewhere, e.g., try to find another physician or go to an emergency room? Choose <u>one</u> option below by placing an "x" in the box that best represents your response.

☐ Today

☐ 2 Days from Now

☐ Seek Care Elsewhere

Figure 2: Sample choice question under urgent health condition.